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A NORMAL APPROXIMATION FOR THE MULTIVARIATE LIKELIHOOD RATIO STATISTICS*

by

Govind S. Mudholkar*
Department of Statistics
University of Rochester
Rochester, N.Y. 14627

and

Madhusudan C. Trivedi
Pennwalt Corporation
Pharmaceutical Division
P.O. Box 1710
Rochester, N.Y. 14603

ABSTRACT

For many multivariate hypotheses, under the normality assumpions, the likelihood ratio tests are optimal in the sense of aving maximal exact slopes. The exact distributions needed for implementing these tests are complex and their tabulation is limited in scope and accessibility. In this paper, a method of constructing normal approximations to these distributions is described, and illustrated using the problems of testing sphericity and independence between two sets of variates. The normal approximations are compared with well known competing approximations and are seen to fare well.

Key Words: Sphericity, independence between two sets of variates.

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1. INTRODUCTION

For most testing of hypothesis problems in multivariate analysis, under the normality assumption, several reasonable solutions of comparable merit exist. These include the tests resulting from the union-intersection principle, the class of likelihood ratio criteria and adhoc statistics such as Bartlett-Pillai trace for MANOVA. The Neyman-Pearson theory provides some information on the operating characteristics of these procedures, but does not indicate any of the contenders as superior. However, as demonstrated by Hsieh (1979), the likelihood ratio tests for many of the multivariate hypotheses have maximal exact slopes, i.e., they are asymptotically optimal according to Bahadur's (1967) method of comparing tests. From a practical standpoint the null distributions of the likelihood ratio statistics or of their competitors, are of crucial importance. These distributions, where available, are complex, their tables are generally limited in scope and not often accessible. Moreover, the tabulations concern only selected percentiles and are inadequate for computing the p-values needed in practice. The pragmatic approach to such distribution problems from early days (e.g., Neyman and Pearson, 1931) is to seek reasonably accurate and convenient approximations to the distributions.

The principal methods of approximating a likelihood ratio use the fact that, in large samples, its distribution is approximately of Pearson type I form and that of its negative logarithm is of type III, i.e., chi-square, form. Nayer (1936) following a suggestion by Neyman and Pearson (1931) used the moments to approxi-

mate the percentiles for testing the homogeneity of variances in this manner. Bishop (1939), on the other hand, obtained empirical expressions for the parameters for a type I approximation by passing the intermediate stage of computing the moments. Bartlett (1937) pursuing the asymptotic chi-square character of a negative multiple of loglikelihood ratio, pointed out by Neyman and Pearson (1931), used moments to approximate it by a scaled chi-square variable for samples of moderate size. This approximation deteriorates as the size of the problem, as measured by the dimension of the multivariate normal distribution or by the number of populations in the problem increases, or when the effective sample size is small. A comprehensive investigation of various approximations was conducted by Box (1949), in which he introduced new widely known and used asymtotic chi-square series approximations for the distributions of likelihood ratios. Box studied his series approximations, in the context of two multivariate problems, comparing them with the exact distributions and with several other approximations including one based on the F-distribution.

The purpose of this essay is to describe a method for constructing a Gaussian approximation to the null distribution of the likelihood ratio, and to demonstrate its efficacy and relevance in testing multivariate hypotheses. The normal approximation is outlined in Section 2. It is illustrated using two common multivariate problems, namely testing independence of two sets of variates and testing the sphericity hypotheses. Section 3 contains the likelihood ratio statistics for the two problems together

with current approximations for their null distributions. These approximations are then numerically compared with the new normal approximation in Section 4.

2. A NORMAL APPROXIMATION FOR THE LIKELIHOOD RATIO Λ

Let $Y_1, Y_2, \ldots, Y_n$ be a sequence of asymptotically normally distributed nonnegative random variables. The convergence of the distribution of Y_n to normality can be accelerated by approximately symmetrizing it with a transformation as follows:

Let $\kappa_r = \kappa_r(n)$, $r = 1, 2, \ldots$, denote the cumulants of $Y = Y_n$ and suppose that $\kappa_1 \to \infty$ and $\kappa_r/\kappa_1 = \phi_r$, $r \geq 2$, are bounded as $n \to \infty$. Then using the Taylor series it is easy to obtain the following asymptotic expansion for the expectation $E(Y/\kappa_1)^h$ of a power of Y as

$$\mu_1'(h) = 1 + \frac{h(h-1)\phi_2}{2\kappa_1} + \frac{h(h-1)(h-2)}{24\kappa_1^2}\{4\phi_3 + 3(h-3)\phi_2^2\} + O(\kappa_1^{-3}). \tag{1}$$

From this the r^{th} moment of $(Y/\kappa_1)^h$ can be obtained by substituting (rh) for h in (1). The following central moments of $(Y/\kappa_1)^h$ are then obtained in a routine manner:

$$\mu_2(h) = \frac{h^2\phi_2}{\kappa_1} + \frac{h^2(h-1)}{2\kappa_1^2}\{2\phi_3 + (3h-5)\phi_2^2\} + O(\kappa_1^{-3}), \tag{2}$$

$$\mu_3(h) = \frac{h^3}{\kappa_1^2}\{\phi_3 + 3(h-1)\phi_2^2\} + O(\kappa_1^{-3}), \tag{3}$$

$$\mu_4(h) = \frac{3h^4\phi_2^2}{\kappa_1^2} + O(\kappa_1^{-3}).$$

Since Y is asymptotically normally distributed as $n\to\infty$, by the Mann-Wald (1943) theorem so is an appropriately normalized $(Y/\kappa_1)^h$. This convergence to normality is accelerated if h is chosen so that the leading term in the expansion (3) for $\mu_3(h)$ vanishes. This value h_o of h which approximately symmetrizes $(Y/\kappa_1)^h$ is obtained from (3) as

$$h_o = 1 - \kappa_1\kappa_3/(3\kappa_2^2).$$

The distribution of $(Y/\kappa_1)^{h_o}$ may be approximated by the normal distribution with mean $\mu_1'(h_o)$ and variance $\mu_2(h_o)$ given in (1) and (2), respectively. That is,

$$\Pr(Y \leq y) \simeq \Phi[\{(y/\kappa_1)^{h_o} - \mu_1'(h_o)\}/\sigma(h_o)], \tag{4}$$

where $\sigma^2(h_o) = \mu_2(h_o)$ is given by (2).

It is well known, e.g., see Anderson (1958) or Srivastava and Khatri (1979), that for many likelihood ratio statistics Λ appearing in multivariate analysis under the normality assumption, $U = \Lambda^{2/N}$ is distributed as a product ΠX_i of independent beta variates X_i, $i = 1, 2, \ldots k$, distributed according to $B(X_i;\, a_i,\, b_i)$, where N is the number of observations. Equivalently, we have $-\log U = \sum_{i=1}^{k} (-\log X_i)$ in distribution. Now, it can be shown that, as a_i and $b_i\to\infty$, $-\log X_i$ converges in law to normality. Hence, it is possible to construct a normal approximation for U as described above. Towards this end we need the cumulants of $-\log X_i$. The moment generating function of $-\log X_i$ is easily seen to be $M(t) = B(a_i-t,b_i)/B(a_i,b_i)$. Hence, the cumulant generating function is

$$K(t) = \log\{\Gamma(a_i+b_i)/\Gamma(a_i)\} - \log\{\Gamma(a_i+b_i-t)/\Gamma(a_i-t)\}.$$

Differentiating and using $\Psi(Z) = \frac{d}{dZ} \log \Gamma(Z)$, the r^{th} cumulant of $-\log X_i$ is

$$C_{ri} = (-1)^r\{\Psi^{(r-1)}(a_i) - \Psi^{(r-1)}(a_i+b_i)\}.$$

But $\Psi'(Z) = -\sum_{j=0}^{\infty} (Z+j)^{-1}$, giving

$$C_{ri} = (r-1)![\sum_{j=0}^{m-1} (a_i+j)^{-r} + \sum_{j=m}^{\infty} \{(a_i+j)^{-r} - (a_i+j+v)^{-r}\}], \quad (5)$$

where m denotes the largest integer in b_i and $v = b_i - m$. The cumulants of $-\log U'$ obtained using (5) are,

$$\kappa_r(U') = (r-1)![\sum_{i=1}^{k} \sum_{j=0}^{m-1} (a_i+j)^{-r}$$
$$+ \sum_{i=1}^{k} \sum_{j=m}^{\infty} \{(a_i+j)^{-r} - (a_i+j+v)^{-r}\}]. \quad (6)$$

If b_i is an integer then the second sum in (6) vanishes and

$$\kappa_r(U') = (r-1)! \sum_{i-1}^{k} \sum_{j=0}^{m-1} (a_i+j)^{-r}. \quad (7)$$

From (7) we observe that as either k or b_i or both $\to\infty$, κ_1 diverges, but κ_r, $r > 1$, are bounded. That is, $\phi_r = \kappa_r/\kappa_1 \to 0$. Hence, it is possible to construct the normal approximation to the distribution of Λ as described above. Thus, from (4) we get

$$Pr(\Lambda \geq \lambda) \simeq \Phi[\{(\lambda'/\kappa_1)^{h_o} - \mu_1'(h_o)\}/\sigma(h_o)], \quad (8)$$

where $\lambda' = -2(\log \lambda)/N$. The $100(1-\alpha)^{th}$ percentile $\Lambda_{1-\alpha}$ can be approximated as

$$\Lambda_{1-\alpha} \simeq \kappa_1\{Z_\alpha\sigma(h_o) + \mu_1'(h_o)\}^{1/h_o}, \quad (9)$$

where Z_α denotes the $100\alpha^{th}$ percentile of the standard normal variate.

3. TWO APPLICATIONS IN MULTIVARIATE ANALYSIS

The normal approximation derived in the previous section is now illustrated and later examined in the context of the multivariate problems of testing independence between two sets of normal variates and testing the sphericity hypothesis.

3.1 Independence Between Two Sets. Let $\underset{\sim}{X}' = (X_1, X_2, \ldots, X_{p_1})$ and $\underset{\sim}{Y}' = (Y_1, Y_2, \ldots, Y_{p_2})$, $p_1 < p_2$, $p_1 + p_2 = p$, be jointly normally distributed with $\mathrm{Var}(\underset{\sim}{X}) = \Sigma_{11}$, $\mathrm{Var}(\underset{\sim}{Y}) = \Sigma_{22}$ and $\mathrm{Cov}(\underset{\sim}{X},\underset{\sim}{Y}) = \Sigma_{12}$. The hypothesis of independence between $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ is $H_o:\Sigma_{12} = 0$. If S is the usual estimate of Σ based on a sample of size N, then the likelihood ratio statistic for testing H_o is

$$\Lambda = [|S|/(|S_{11}||S_{22}|)]^{N/2},$$

where S_{11} and S_{22} are the submatrices of S corresponding to Σ_{11} and Σ_{22}, respectively. The exact distribution of the statistic Λ is given, and tabulated for some values of p_1 and p_2, by several authors (e.g., see Krishnaiah 1979 and Consul 1967a). Among various approximations proposed for the null distribution of Λ two are well known and widely used in statistical packages such as BMDP (see Engelman, et al., 1977). These are (i) the chi-square series approximation due to Box (1949) and (ii) the F approximation due to Rao (1948).

Box-Approximation. Let $w = p_1p_2$, $m = N - (p_1+p_2+3)/2$, $\gamma_2 = w(p_1^2+p_2^2-5)/48$, $\gamma_4 = \gamma_2^2/2 + w\{3(p_1^4+p_2^4) + 10w^2 - 50(p_1^2+p_2^2)$

$+ 159\}/1920$. Then,

$$\begin{aligned}
&\Pr(-m \log U \le z) \\
&\quad \simeq \Pr(\chi_w^2 \le z) + \gamma_2\{\Pr(\chi_{w+4}^2 \le z) - \Pr(\chi_w^2 \le z)\}/m^2 \\
&\quad + [\gamma_4\{\Pr(\chi_{w+8}^2 \le z) - \Pr(\chi_w^2 \le z)\} - \gamma_2^2\{\Pr(\chi_{w+4}^2 \le z) \\
&\quad - \Pr(\chi_w^2 \le z)\}]/m^4 + O(N^{-6}), \qquad (10)
\end{aligned}$$

where χ_k^2 denotes a chi-square variable with k degrees of freedom and $U = \Lambda^{2/N}$.

Rao-Approximation. Let $m' = N - (p_1+p_2+3)/2$, $L = (p_1p_2 - 2)/4$, $s = \surd\{(p_1^2p_2^2 - 4)/(p_1^2+p_2^2 - 5)\}$. Then,

$$Q = (m's - 2L)(1 - U^{1/s})/(p_1p_2U^{1/s}), \qquad (11)$$

has an F-distribution with p_1p_2 and $m's - 2L$ degrees of freedom.

Now, it is well known that (e.g., see Anderson, 1958, p. 236) under H_o the likelihood ratio statistic Λ satisfies the equivalence $\Lambda^{2/N} = U = \Pi X_i$ in law, where $X_i (i = 1, 2, \ldots, p_2)$ are independently distributed according to beta distributions $B\{X_i; (N - p_1 - i)/2, p_1/2\}$. The normal approximation developed in the previous section can be specialized in this case by taking $k = p_2$, $a_i = (N - p_1 - i)/2$, $b_i = p_1/2$ in the expressions (6) for the cumulants, (8) for the probabilities, and (9) for the percentiles of Λ.

3.2 Testing the Sphericity Hypothesis. Let $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \ldots, \underset{\sim}{X}_N$ be a random sample from a p-variate normal population with mean $\underset{\sim}{\mu}$ and covariance matrix Σ. The hypothesis that the p components of the random vector $\underset{\sim}{X}$ are independent with the same variance i.e., $H_o: \Sigma = \sigma^2 I_p$, $\sigma^2 > 0$ unknown, is known as the sphericity hypothesis. The hypothesis also arises in the analysis of data from experiments

consisting of repeated measurements. In these experiments, the measurements on a subject are assumed to have compound symmetry, i.e., have the same variances and same correlations. The problem of testing the hypothesis of compound symmetry $H_o:\Sigma = \sigma^2(\rho I + (1-\rho)I)$ for the covariance structure of $(p+1)$ repeated measurements $\underset{\sim}{Y}$ can be reduced to the sphericity hypothesis by an orthogonal transformation $\underset{\sim}{Y}' \rightarrow \underset{\sim}{Y}'(\underset{\sim}{1}/\sqrt{(p+1)}:T_1)$ where $\underset{\sim}{1}$ is the vector of 1's. $\underset{\sim}{Y}$ satisfies compound symmetry if and only if $\underset{\sim}{X} = T_1\underset{\sim}{Y}$ satisfies the sphericity hypothesis. The likelihood ratio criterion for the sphericity hypothesis was proposed by Mauchly (1940) as

$$U = \Lambda^{2/N} = |S|\{(trS)/p\}^{-p},$$

where S is the covariance matrix of the sample of size N. He also derived its null distribution for $p = 2$. The exact null distribution of U for $p = 3$, 4 and 6 was obtained by Consul (1967b). The 5% and 1% points for $p = 4(1)10$ were given by Nagarsanker and Pillai (1973). The series approximation due to Box can be expressed in this case as follows.

<u>Box-Approximation</u>. Let $e = p(p+1)/2 - 1$, $f = n - (2p^2 + p+2)/(6p)$ and $g = (p+2)(p-1)(p-2)(2p^3+6p^2+3p+2)/(288p^2)$ for $n = N-1$. Then,

$$\Pr(- f \log U \leq z) \simeq \Pr(\chi_e^2 \leq z) + g\{\Pr(\chi_{e+4}^2 \leq z) - \Pr(\chi_e^2 \leq z)\}/f^2 + O(f^{-3}). \tag{12}$$

It is well known (e.g. see Srivastava and Khatri, 1979) that the distribution of U under H_o is the same as that of the product ΠX_i, where X_i $(i = 1, 2, \ldots, p-1)$ are independent beta random

variables distributed according to $B\{x_i; (n-i)/2, i(p+2)/(2p)\}$, $n = N - 1$. Again, we can obtain the cumulants of $U' = -2(\log U)/N$ using (6) with $a_i = (n-i)/2$, $b_i = i(p+2)/(2p)$ and $k = p - 1$. Hence, the probabilities and the percentiles of the likelihood ratio Λ may be obtained from (8) and (9), respectively.

4. NUMERICAL COMPARISONS

The quality of the normal approximations for the two multivariate likelihood ratio statistics discussed in the previous section and the other two approximations, was examined by computing the probabilities corresponding to the tabulated percentiles of the statistics. Thus, in the case of the null distribution of Λ for testing independence, the approximations due to Box (10), due to Rao (11), and the normal approximation given in Section (3.1) were used to compute the probabilities corresponding to <u>all</u> 5% and 1% points of Λ given in Pearson and Hartley (1972, p. 99 and 333). Similarly, in case of the sphericity problem, all percentiles given by Nagarsanker and Pillai (1973) were used to examine the approximation due to Box given by (12) and the relevant normal approximation. In both cases, the series approximation due to Box was used in two steps: 1) only the first term; and 2) all terms given in (10) and (12). Also the percentiles approximated using the normal approximations were compared with the competing approximations using the first term of the Box series and the F-approximation. A selection of errors, i.e., (Approximation - Exact value)X10^5, in various cases is presented in Tables 1 and 2.

Conclusions. Let New, Rao, Box 1 and Box 3 denote the normal approximation, the F-approximation due to Rao, the first term approximation due to Box and the three term approximation due to Box, respectively. From Tables 1 and 2 it may be observed that (i) Rao, Box 1 and Box 3 have errors in second through fifth decimal place, they are especially large for small N and decreasing rapidly as N increases. The normal approximation has errors in fourth or fifth decimal place. (ii) As p_1, p_2 or p increases, errors due to Rao, Box 1 and Box 3 increase while those due to the normal approximation either decrease or maintain the same level. Overall, the normal approximation is superior for small N and is comparable with the others when the N is large.

TABLE 1. Errors of the Approximations for the Likelihood Ratio Statistic for Testing Independence Between Two Sets

				α = .05						
P1	P2	N	λ	ERRORS*						
				PERCENTILES			PROBABILITIES			
				NEW	RAO	BOX1	NEW	RAO	BOX1	BOX3
3	8	12	0.00001	0	0	67	-19	-1537	-4991	-4744
	8	19	0.04107	4	7	698	-17	-24	-1776	-50
	22	30	0.00107	0	1	356	0	-143	-4936	-3893
	22	37	0.01620	0	0	760	-4	0	-3842	-725
5	8	18	0.00217	0	5	161	2	-218	-3340	-485
	8	25	0.03411	1	3	352	-8	-17	-1355	-16
	16	26	0.00019	0	0	50	12	-420	-4759	-2732
	16	33	0.00568	0	1	213	-6	-48	-3160	-316
7	8	19	0.00021	0	1	37	-9	-618	-4225	-1437
	8	23	0.00356	0	3	133	-3	-122	-2657	-199
	10	21	0.00007	0	0	18	25	-746	-4531	-2059
	10	25	0.00157	0	2	81	9	-155	-3153	-358
				α = .01						
3	8	12	0.00000	0	0	17	6	-478	-999	-997
	8	19	0.02261	0	7	514	0	-9	-487	-29
	22	30	0.00043	0	0	209	0	-44	-997	-935
	22	37	0.00990	-2	-1	572	6	5	-872	-303
5	8	18	0.00085	0	3	85	3	-72	-803	-223
	8	25	0.02086	0	6	270	-2	-11	-374	-15
	16	26	0.00007	0	0	25	-10	-143	-985	-784
	16	33	0.00327	0	0	147	3	-11	-753	-147
7	8	19	0.00007	0	0	16	1	-192	-932	-513
	8	23	0.00174	0	2	81	-1	-42	-665	-102
	10	21	0.00002	0	0	8	3	-234	-965	-659
	10	25	0.00075	0	1	48	5	-48	-756	-164

TABLE 2. Errors of Approximations for the Likelihood Ratio Statistic for Testing Sphericity

			α = .05						α = .01				
P	N	λ	ERRORS*					λ	ERRORS*				
			PERC.		PROB.				PERC.		PROB.		
			NEW	BOX1	NEW	BOX1	BOX2		NEW	BOX1	NEW	BOX1	BOX2
4	10	0.09739	16	467	-21	-539	-57	0.05010	-22	382	10	-173	-25
	15	0.25350	35	236	-29	-190	-10	0.17210	-41	267	9	-62	-4
	20	0.37720	43	120	-33	-97	-5	0.28670	-44	166	8	-31	-1
	30	0.53900	46	37	-38	-40	-2	0.45310	-38	76	7	-12	0
5	10	0.03110	1	444	-7	-1171	-236	0.01361	-5	281	7	-350	-96
	15	0.13780	7	334	-10	-393	-29	0.08685	-19	287	7	-124	-14
	20	0.24820	17	201	-17	-194	-7	0.17970	-23	188	6	-63	-5
	30	0.41640	28	79	-27	-78	-2	0.34020	-29	58	6	-23	0
7	10	0.00094	0	104	20	-3352	-1789	0.00025	0	43	2	-822	-566
	15	0.02712	0	297	-4	-1158	-228	0.01444	-3	207	4	-330	-86
	20	0.08446	5	282	-12	-562	-63	0.05514	-5	234	3	-165	-24
	30	0.21780	15	163	-21	-216	-13	0.16770	-4	153	1	-65	-5

*Error in prob. = (Approx. value − α)x10^5 and in perc. (Approx. value − λ)x10^5.

REFERENCES

Anderson, T. W. (1958). *An Introduction to Multivariate Statistical Analysis*. John Wiley and Sons, New York.

Bahadur, R. R. (1967). An optimal property of the likelihood ratio statistics. *Proceedings of Fifth Berkley Symposium Mathematical Statistics and Probability*. Univ. of California Press, 13-26.

Bartlett, M. S. (1937). Properties of sufficiency and statistical tests. *Proceedings of Royal Statistical Society, A*, **160**, 268.

Bishop, D. J. (1939). On the comprehensive test of the homogeneity of variances and covariances in multivariate problems. *Biometrika*, **31**, 31-55.

Box, G. E. P. (1949). A general distribution theory for a class of likelihood criteria. *Biometrika*, **36**, 317-46.

Consul, P. C. (1967a). On the exact distribution of likelihood ratio criteria for testing independence of sets of variates under the null hypothesis. *Annals of Mathematical Statistics*, **38**, 1160-9.

Consul, P. C. (1967b). On the exact distribution of the criterion W for testing sphericity in a p-variate normal distribution. *Annals of Mathematical Statistics*, **38**, 1170-4.

Engelman, L., Frane, J. W., and Jennrich, R. I. (1977). *Biomedical Computer Programs P-Series*. Univ. of California Press, Berkley, California.

Hsieh, H. K. (1979). On asymptotic optimality of likelihood ratio tests for multivariate normal distributions. *Annals of Statistics*, **7**, 592-8.

Krishnaiah, P. R. (1979). *Some Recent Developments on The Real Multivariate Distributions*. North Holland Co., New York.

Mann, H. B. and Wald, A. (1943). On stochestic limit and order relationships. *Annals of Mathematical Statistics*, **14**, 217-26.

Mauchly, J. W. (1940). Significance test for sphericity of a normal n-variate distribution. *Annals of Mathematical Statistics*, **11**, 204-9.

Nagarsanker, B. N. and Pillai, K. C. S. (1973). Distribution of the sphericity test criterion. *Journal of Multivariate Analysis*, **3**, 226-35.

Nayer, P. P. N. (1936). Statistical Research Memorandum, 1, 38.

Neyman, J. and Pearson, E. S. (1931). Bulletin International Academy Cracovie, A, 460.

Pearson, E. S. and Hartley, H. O. (1972). Biometrika Tables for Statisticians, 2. Cambridge Univ. Press, New York

Rao, C. R. (1948). Tests of significance in multivariate analysis. Biometrika, 35, 58-79.

Srivastava, M. S. and Khatri, C. G. (1979). An Introduction to Multivariate Statistics, North Holland Co., New York

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FOR MANY MULTIVARIATE HYPOTHESES, UNDER THE NORMALITY ASSUMPTIONS, THE LIKELIHOOD RATIO TESTS ARE OPTIMAL IN THE SENSE OF HAVING MAXIMAL EXACT SLOPES. THE EXACT DISTRIBUTIONS NEEDED FOR IMPLEMENTING THESE TESTS ARE COMPLEX AND THEIR TABULAITON IS LIMITED IN SCOPE AND ACCESSIBILITY. IN THIS PAPER, A METHOD OF CONSTRUCTING NORMAL APPROXIMATIONS TO THESE DISTRIBUTIONS IS DESCRIBED, AND ILLUSTRATED USING THE PROBLEMS OF TESTING SPHERICITY AND INDEPENDENCE BETWEEN TWO SETS OF VARIATES. THE NORMAL APPROXIMATIONS ARE COMPARED WITH WELL KNOWN COMPETING APPROXIMATIONS AND SEEN TO FARE WELL.

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